From Combinatorics and Probability to Statistics

a. The Binomial Theorem and Pascal's Triangle,
b. The Traveling Salesmen Problem,
c. Mendel's Laws,
d. The Law of Addition and the Law of Multiplication,
e. The Hypergeometric and Binomial Distributions and Vandermonde's Identity, and
f. The P-Value.

Problem 1 (32%):
Consider a population of the fruit fly *Drosophila melanogaster*.
Assume that in the first generation the female genotype is XX and the male genotype is XY.

a) What is the probability for a female offspring?
b) What is the probability for a male offspring?
c) What is the probability that six offspring will be females?
d) How many different possibilities are there to select five of six offspring?
e) What is the probability that five of six offspring will be females?
f) How many different possibilities are there to select four/three/two/one of six offspring?
g) What is the probability that four/three/two/one of six offspring will be females?
h) What is the sum of the probabilities of Problems 1(c), 1(e) and 1(g)?
i) What event is missing in this sum that will make the sum 1?
j) What theorem proves that accounting for this event will make the sum of probabilities 1?
k) Use the theorem to prove that accounting for this event will make the sum of probabilities 1.
l) What row in Pascal's triangle relates to this list of events of probability 1?
m) What is the probability that at least five of six offspring will be females?
n) A researcher observed one male in a group of six offspring. What is the p-value for this event?
o) A researcher observed one male in a group of 60 offspring. What is the p-value for this event?
p) Design an experiment that tests the assumption.

Problem 2 (18%):
Consider a population of the fruit fly *Drosophila melanogaster*.
Assume that in the first generation the female and male genotypes are Aa, where A and a denote an autosomal chromosome that carries either a dominant or a recessive allele of a gene determining the color of the fly, with dominant being yellow and recessive being green.

a) What is the probability for a green offspring?
b) What is the probability for a yellow offspring?
c) What is the probability that six offspring will be green?
d) What is the probability that five of six offspring will be yellow?
e) What is the probability that at least five of six offspring will be green?
f) A researcher observed one yellow fly in a group of six offspring. What is the p-value for this event?
g) A researcher observed two yellow flies in a group of 60 offspring. What is the p-value for this event?
h) Prepare a table of the expected joint segregation of color and sex in the second-generation population.
i) Design an experiment that measures the numbers in the table and tests the assumption.

Problem 3 (18%): X-Chromosome Linkage
Consider a population of the fruit fly *Drosophila melanogaster*.
Assume that in the first generation the female genotype is xX and the male genotype is xY, where the x-chromosome carries a recessive allele of the fly's color (green), and the X-chromosome carries a dominant allele of that gene (yellow).

a) What is the probability for a female offspring?
b) What is the probability for a male offspring?
c) What is the probability for a green offspring?
d) What is the probability for a yellow offspring?
e) What is the conditional probability that a female/male offspring will be green/yellow?
f) What is the conditional probability that a green/yellow offspring will be male/female?
g) Prepare a table of the expected joint segregation of color and sex in the second-generation population.

Problem 4 (32%): Sex-Chromosome Linkage
Consider a population of the fruit fly *Drosophila melanogaster*.
Assume that in the first generation the female genotype is Xx and the male genotype is Xy, where the x- and y-chromosomes carry a recessive allele of the fly's color (green), and the X- and Y-chromosomes carry a dominant allele of that gene (yellow).

a) Prepare a table of the expected joint segregation of color and sex in the second-generation population.
b) Design an experiment that compares the assumption above to that of Problem 3.