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(* BIOEN 3070/6070: Introduction to Statistics for Bioengineers *)
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(* Assignment 4: Simulations of the Binomial Distribution *)

(* The Unbiased Binomial Distribution *)

(* -10%: Please attach a cover page. *)

(* General Commands *)

Clear["Global`*"]

(* The Flip of a Single Unbiased Coin which Sides are "0" and "1" *)

Random[BinomialDistribution[1, 0.5]]
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(* Problem 1 (5%): Explain the code above. *)

? Random
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Random[] gives a uniformly distributed pseudorandom Real in the range 0 to 1.
Random[*type*, *range*] gives a pseudorandom number of the specified type, lying in the specified range. Possible types are: Integer, Real and Complex. The default range is 0 to 1. You can give the range {*min*, *max*} explicitly; a range specification of *max* is equivalent to {0, *max*}. ≫

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? BinomialDistribution
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BinomialDistribution[*n*, *p*] represents a binomial distribution with *n* trials and success probability *p*. ≫

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? Table
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Table[*expr*, {i_{max}}] generates a list of i_{max} copies of *expr*.
Table[*expr*, {*i*, i_{max}}] generates a list of the values of *expr* when *i* runs from 1 to i_{max}.
Table[*expr*, {*i*, i_{min}, i_{max}}] starts with $i = i_{min}$.
Table[*expr*, {*i*, i_{min}, i_{max}, *di*}] uses steps *di*.
Table[*expr*, {*i*, {i_1, i_2, …}}] uses the successive values i_1, i_2, ….
Table[*expr*, {*i*, i_{min}, i_{max}}, {*j*, j_{min}, j_{max}}, …] gives a nested list. The list associated with *i* is outermost. ≫

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(* Problem 2 (5%): Write code that simulates multiple flips of a single
  unbiased coin. Explain your code. Hint: Use the "Table" built-in command. *)

(* Problem 3 (5%): Run your code to simulate (a) two,
(b) 10, and (c) 100 flips of the coin. In each case,
calculate the sample mean -- http://mathworld.wolfram.com/SampleMean.html ,
and the sample variance -- http://mathworld.wolfram.com/
  SampleVariance.html . Hint: Do not use the built-in "Variance" command as is. *)

(* Problem 4 (10%): What is the probability distribution that you
  are sampling? Write its analytical expression explicitly for (a) two,
(b) 10, and (c) 100 flips of the coins. *)
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(* The Simultaneous Flip of Identical Unbiased Coins *)
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Random[BinomialDistribution[10, 0.5]]
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(* Problem 5 (5%): Explain the code above. *)
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(* Problem 6 (5%):
 Run this code and its modifications to simulate the simultaneous flip of (a) two,
(b) 10, and (c) 100 identical unbiased coins. In each case,
calculate the sample mean and the sample variance. *)
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(* Problem 7 (5%): What is the probability distribution that you are sampling? Write
  its analytical expression explicitly for the simultaneous flip of (a) two,
(b) 10, and (c) 100 identical unbiased coins. *)
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(* Problem 8 (5%):
 Based on the probability distribution you explicitly wrote in Problems 4 and 7,
would you expect a different or a similar sample mean in the
 case of 10 flips of one unbiased coin in comparison with the
 case of one flip of 10 identical unbiased coins? Explain. *)
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(* Problem 9 (5%):
 Write code that simulates multiple simultaneous flips of 10 identical unbiased
  coins. Explain your code. Test your code by running (a) two, (b) 10,
and (c) 100 simultaneous flips of 10 coins. Hint: Use the "Table" built-in command. *)
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? Count
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Count[*list*, *pattern*] gives the number of elements in *list* that match *pattern*.
Count[*expr*, *pattern*, *levelspec*] gives the total number of subexpressions matching *pattern* that appear at the levels in *expr* specified by *levelspec*. ≫

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(* Problem 10 (10%): Define an event as observing 3 times the "1" side of a coin --
  and therefore 7 times the "0" side of a coin -- in a single simultaneous
  flip of 10 identical unbiased coins. Write code that counts the number of
  such events observed in the n simultaneous flips of 10 coins. Explain your
  code. Test your code by counting the number of such events in (a) two, (b) 10,
and (c) 100 simultaneous flips of 10 coins. Hint: Use the "Count" built-in command. *)
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(* Problem 11 (5%):
 What are all the possible events in a single simultaneous flip of 10 coins? *)
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(* Problem 12 (5%): Write code that counts the number of each of the events you
  identified in Problem 11 as observed in the n simultaneous flips of 10 coins. Explain
  your code. Test your code by counting the numbers of the events in (a) two, (b) 10,
and (c) 100 simultaneous flips of 10 coins. Hint: Use the "Table" built-in command. *)
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(* Problem 13 (5%): Modify the code of Problem 12 to calculate the observed
  frequency of each event instead of the observed number. Explain your
  code. Test your code by calculating the frequencies of the events in (a) two,
(b) 10, and (c) 100 simultaneous flips of 10 coins. *)
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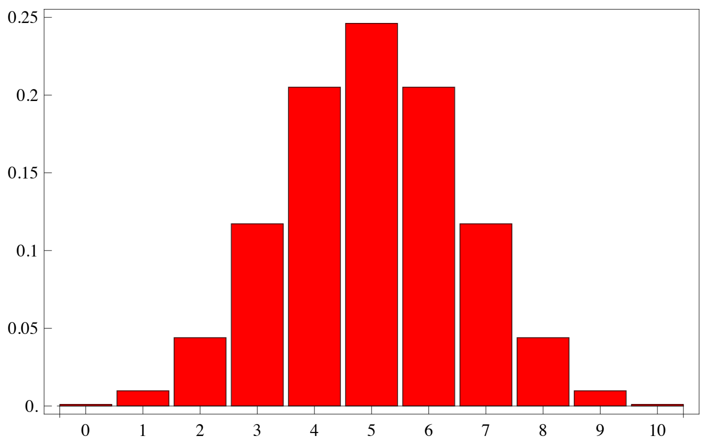
f[k_] := Binomial[10, k] * 0.5^k * (1 - 0.5) ^ (10 - k);
theoreticalFrequencies = Table[f[a], {a, 0, 10}];
Sum[theoreticalFrequencies[[a]], {a, 1, 11}];
BarChart[theoreticalFrequencies,
 PlotRange → {{0.5, 11.5}, {0, 0.25}},
 ChartStyle → Red,
 Frame → True,
 FrameTicks → {
   Table[{a + 1, ToString[a]}, {a, 0, 10}],
   Table[a, {a, 0, 0.25, 0.05}],
   None,
   None}]
```

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(* Problem 14 (10%):
 Explain the code above. What does the variable "theoreticalFrequencies" describe? *)

(* Problem 15 (15%):
 Modify the code above to plot the frequencies of the events in (a) two,
(b) 10, and (c) 100, (d) 10^3, (e) 10^4, (f) 10^5 simultaneous flips
 of 10 coins. Compare the plots to the one above and explain. *)
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(* Sampling a Distribution *)
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(* Problem 16 (10% Extra Credit): In these simulation experiments,
an emphasis is made on the coins being identical. Why? *)
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(* The Biased Binomial Distribution *)
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(* Problem 17 (15% Extra Credit): Modify the code of Problem 14 to plot the
  theoretical frequencies of the simultaneous flip of 10 identical biased coins,
where the probability to observe the "1" side of the coin is (a) 0.1,
(b) 0.05, and (c) 0.01. Compare the plots to the one above. *)

(* Problem 18 (15% Extra Credit):
 Write and run code that simulates the simultaneous flip of (a) two, (b) 10,
and (c) 100 identical biased coins where the probability to observe the
 "1" side of the coin is 0.05, that is, a magnitude smaller than 0.5. *)
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(* Problem 19 (20% Extra Credit):
 Write code that counts the number of each of the possible events in
  the n simultaneous flips of 10 identical biased coins. Explain your
  code. Test your code by counting the numbers of the events in (a) two,
(b) 10, and (c) 100 simultaneous flips of 10 identical biased coins,
where the probability to observe the "1" side of the coin is 0.05. *)

(* Problem 20 (20% Extra Credit):
 Modify the code of Problem 18 to calculate the observed frequency
  of each event instead of the observed number. Explain your code. Test
  your code by calculating the frequencies of the events in (a) two,
(b) 10, and (c) 100 simultaneous flips of 10 identical biased coins,
where the probability to observe the "1" side of the coin is 0.05. *)

(* Problem 21 (20% Extra Credit):
 Plot the frequencies of the events in (a) two, (b) 10, and (c) 100,
(d) 10^3, (e) 10^4, (f) 10^5 simultaneous flips of 10 identical biased coins,
where the probability to observe the "1" side of the coin is 0.05. Compare
 the plots to the one you plotted in Problem 16(b) and explain. *)
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