Combinatorics

Please attach a cover page (-10%).

Problem 1 (100%):
Expressing the binomial coefficients in terms of factorials and then performing whatever simplifications are necessary, prove that:

a) $\binom{n}{k} = \binom{n}{n-k}$;

b) $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$, that is, Pascal's Identity;

c) $\binom{n}{k} = \frac{n}{n-k}\binom{n-1}{k}$.

From J. E. Freund (Dover, 1973), p. 32.

Problem 2 (Extra Credit 100%):
Prove Vandermonde's Identity:

$$\binom{m+n}{k} = \sum_{i=0}^{k}\binom{m}{i}\binom{n}{k-i}$$

a) by formulating the left- and right-sides of the identity as approaches to solving the same combinatorial problem, that is, by using a combinatorial proof;

b) by using the Binomial Theorem:

$$(x+y)^n = \sum_{k=0}^{n}\binom{n}{k}x^k y^{n-k}$$

on both the left- and right-sides of the equation:

$$(1+x)^{m+n} = (1+x)^m(1+x)^n \ .$$