Inhibition of the Measurement of the Wave Function of a Single Quantum System in Repeated Weak Quantum Nondemolition Measurements

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(Received 14 July 1994; revised manuscript received 11 January 1995)

It is shown that a series of repeated weak quantum nondemolition measurements performed on a single quantum system gives no information about the wave function of the system. The physical explanation, based on the quantum Brownian motion and the continuous collapse of the wave function which originate in the projection postulate, is discussed in two specific examples.

PACS numbers: 03.65.Bz, 42.50.–p

Two separate structures exist in quantum mechanics: the observables, which are represented by operators of the Hilbert space, and the physical systems, which are described by state vectors or wave functions. The wave function is said to have an epistemological meaning, because it contains all the relevant information about the physical system. The result of a precise measurement on a single quantum system is always one of the eigenvalues of the measured observable. After the measurement, the wave function of the measured system collapses to the corresponding eigenstate, according to the projection postulate. There is no one-to-one correspondence between the result of a single measurement and the state of the system before the measurement. In order to measure the initial wave function of the system, one needs to prepare an ensemble of systems with the same wave function and measure them all. The wave function is obtained from the statistics of the results of measurements performed on this ensemble. Recently Aharonov, Anandan, and Vaidman [1,2] suggested that the wave function of a single quantum system could be measured, therefore giving the wave function an ontological significance, i.e., physical reality in its own right. They suggested employing a series of "protective measurements," where an *a priori* knowledge of the wave function enables one to measure this wave function and protect it from changing at the same time. However, with this *a priori* knowledge, one could reproduce the wave function after each measurement for an arbitrarily large number of times, and measure the wave function in the conventional manner. Another recent suggestion, made by Royer [3], is to measure the spin wave function of a single spin-$\frac{1}{2}$ particle using "physically reversible measurements." In this process each measurement would be counteracted by another measurement to restore the initial state of the particle, where no *a priori* knowledge of this state is needed. Only the results of measurements which are performed on the particle in its initial state would be taken into account. Huttner [4] showed, however, that the statistics of these results are independent of the initial spin wave function, and therefore no information could be obtained from a series of physically reversed measurements.

In this Letter, we investigate the possibility of measuring the wave function of a single quantum system with no *a priori* knowledge of the wave function, in order to explore a real ontological meaning of the wave function. We study the case of repeated weak quantum nondemolition (QND) measurements [5,6], for which we can assume that the signal and the probe are in pure states before the measurement, without loss of generality. In this case, the signal is left in a pure state after the measurement. The QND observable evades backaction noise in the measurement process and remains unchanged during the time evolution of the signal. In addition, the QND measurement can be chosen to be as weak as we want. It is possible, therefore, to measure the signal many times, using weak QND measurements, before the wave function of the signal is changed significantly. The measurement results are all generated under some influence of the initial wave function, and one may expect the statistics of these results to give at least partial information about this wave function. By "information about the wave function" we mean information about both the average and the variance of the measured observable, i.e., the center and the width of the wave function, with finite probability errors. Information about the center alone corresponds to a measurement of the observable, where information about the width reveals the wave function. In this work, we show that this intuitive picture fails, and one cannot extract any information about the initial wave function of the signal at all using repeated weak QND measurements.

First, we describe the general formulation of this problem. We show that the inhibition of the measurement of the wave function originates in the projection postulate. To illustrate this result, we discuss two specific examples, both using QND measurements which have been demonstrated experimentally. The first example [7] is that of repeated photon number QND measurements [8–11] performed on a squeezed state of light, i.e., a generalized minimum uncertainty wave packet. Note that the photon number QND measurement is analogous to the QND measurement of the momentum of a free particle [12]. The second example is that of alternating QND measurements of the two (slowly varying) quadrature amplitudes [13,14]

 0031-9007/95/74(21)/4106(4)$06.00

of a squeezed state. These measurements are mathematically equivalent to the QND measurements of the position and the momentum of a mechanical harmonic oscillator. Our two examples, therefore, cover all QND measurements known today.

In our general model a series of alternating QND measurements of the two conjugate observables, $\hat{q}$ and $\hat{p}$, is performed on a single quantum system, whose initial state is described by the density operator $\hat{\rho}_0$. The statistics of the $\hat{q}$ measurement results are expected to give information about the initial probability density of $\hat{q}$, $P_0(q) = \langle q|\hat{\rho}_0|q\rangle$, i.e., estimates of the initial center, $\langle q_0\rangle = \int dq\, P_0(q)q$, and the initial width, $\langle \Delta q_0^2\rangle = \langle q_0^2\rangle - \langle q_0\rangle^2$, where $\langle q_0^2\rangle = \int dq\, P_0(q)q^2$. In the same way, the statistics of the $\hat{p}$ measurement results are expected to give information about $P_0(p) = \langle p|\hat{\rho}|p\rangle$. Note that this model applies to the case of repeated measurements of the observable $\hat{q}\cos\theta + \hat{p}\sin\theta$, where θ is an arbitrary parameter. Indeed, one needs, at least, information about all such observables in order to determine the wave function of the measured system.

Let the first measurement be a QND measurement of $\hat{q}$. The signal is correlated to a probe, after which the probe is measured to yield the inferred measurement result $\tilde{q}_1$. The probability-amplitude operator, $\hat{Y}(\hat{q},\tilde{q}_1) = {}_p\langle\tilde{q}_1|\hat{U}(\hat{q})|\psi\rangle_p$, completely describes the three stages of this measurement [15]: the preparation of the probe state, $|\psi\rangle_p$, the interaction of this state with the signal, $\hat{U}(\hat{q})$, and the result of the measurement, $\tilde{q}_1$, which corresponds to the state of the probe after the measurement, $|\tilde{q}_1\rangle_p$. The probability of obtaining the measurement result $\tilde{q}_1$ is

$$P(\tilde{q}_1) = \mathrm{Tr}_s[\hat{X}(\hat{q},\tilde{q}_1)\hat{\rho}_0] = \int dq\, X(q,\tilde{q}_1)P_0(q)\,, \quad (1)$$

where $\hat{X}(\hat{q},\tilde{q}_1) = \hat{Y}^\dagger(\hat{q},\tilde{q}_1)\hat{Y}(\hat{q},\tilde{q}_1)$ is the generalized projection operator, and $X(q,\tilde{q}_1) = {}_s\langle q|\hat{X}(\hat{q},\tilde{q}_1)|q\rangle_s$ is the probability for the probe to undergo a transition when the signal is in the state $|q\rangle_s$. All measurement processes have to satisfy three general requirements. The transition probability of the probe has to be normalized over all possible final states of the probe,

$$\int d\tilde{q}_1\, X(q,\tilde{q}_1) = 1\,. \quad (2)$$

As the inferred value of $\hat{q}$, $\tilde{q}_1$ should equal, on average, the center of the probability density of $\hat{q}$, $\langle\tilde{q}_1\rangle = \int d\tilde{q}_1\, P(\tilde{q}_1)\tilde{q}_1 = \langle q_0\rangle$. This leads to

$$\int d\tilde{q}_1\, X(q,\tilde{q}_1)\tilde{q}_1 = q\,. \quad (3)$$

The signal and the probe should be independent of each other. Therefore, the probability error associated with the measurement result should equal the sum of the measurement error, Δ_m^2, and the intrinsic uncertainty of the wave function, $\langle\Delta\tilde{q}_1^2\rangle = \langle\tilde{q}_1^2\rangle - \langle\tilde{q}_1\rangle^2 = \langle\Delta q_0^2\rangle + \Delta_m^2$, where $\langle\tilde{q}_1^2\rangle = \int d\tilde{q}_1\, P(\tilde{q}_1)\tilde{q}_1^2$. From this we obtain

$$\int d\tilde{q}_1\, X(q,\tilde{q}_1)\tilde{q}_1^2 = q^2 + \Delta_m^2\,. \quad (4)$$

The measurement strength is defined as $1/\Delta_m^2$. Note that the measurement is weak when the measurement error is much larger than the initial intrinsic uncertainty, $\Delta_m^2 \gg \langle\Delta q_0^2\rangle$. After this measurement, the signal is described by the density operator $\hat{\rho} = P(\tilde{q}_1)^{-1}\hat{Y}(\hat{q},\tilde{q}_1)\hat{\rho}_0\hat{Y}^\dagger(\hat{q},\tilde{q}_1)$, and the corresponding probability density of $\hat{q}$ is

$$P(q,\tilde{q}_1) = \langle q|\hat{\rho}|q\rangle = P(\tilde{q}_1)^{-1}X(q,\tilde{q}_1)P_0(q)\,. \quad (5)$$

The next measurement is of the conjugate observable $\hat{p}$. This measurement changes the probability density of $\hat{q}$ from $P(q,\tilde{q}_1)$ to $P_1(q,\tilde{q}_1)$. The center is unchanged,

$$\int dq\, P_1(q,\tilde{q}_1)q = \int dq\, P(q,\tilde{q}_1)q\,, \quad (6)$$

but the width increases due to the backaction noise, Δ_b^2,

$$\int dq\, P_1(q,\tilde{q}_1)q^2 = \int dq\, P(q,\tilde{q}_1)q^2 + \Delta_b^2\,. \quad (7)$$

Now $\hat{q}$ is measured for the second time. Following the treatment of the first measurement of $\hat{q}$ in Eqs. (1)–(4), the conditional probability to obtain $\tilde{q}_2$ in this measurement, after $\tilde{q}_1$ is obtained in the previous measurement, is

$$P(\tilde{q}_2\,|\,\tilde{q}_1) = \int dq\, X(q,\tilde{q}_2)P_1(q,\tilde{q}_1)\,. \quad (8)$$

Obviously, each of the measurement results, $\tilde{q}_1$ or $\tilde{q}_2$, estimates the initial center, $\langle q_0\rangle$. Also, one can estimate the second-order moment $\langle q_0^2\rangle$ using either $\tilde{q}_1^2$ or $\tilde{q}_2^2$, since

$$\langle\tilde{q}_1^2\rangle = \int d\tilde{q}_1\, P(\tilde{q}_1)\tilde{q}_1^2 = \langle q_0^2\rangle + \Delta_m^2\,, \quad (9)$$

$$\langle\tilde{q}_2^2\rangle = \int d\tilde{q}_1\, P(\tilde{q}_1)\int d\tilde{q}_2\, P(\tilde{q}_2\,|\,\tilde{q}_1)\tilde{q}_2^2 = \langle q_0^2\rangle + \Delta_m^2 + \Delta_b^2\,. \quad (10)$$

However, one cannot estimate the initial width, $\langle\Delta q_0^2\rangle$, using a single measurement result, because a single result does not contain information about $\langle q_0\rangle^2$. If $\tilde{q}_1$ and $\tilde{q}_2$ were independent results, obtained from two different quantum systems, which are initially in the same quantum state, their correlation would provide the missing information about $\langle q_0\rangle^2$, and $\langle\Delta q_0^2\rangle$ could be estimated using both measurement results. In our case the second result, $\tilde{q}_2$, depends on the first, $\tilde{q}_1$, and their correlation does not give information about $\langle q_0\rangle^2$, rather it gives

$$\langle\tilde{q}_1\tilde{q}_2\rangle = \int d\tilde{q}_1\, P(\tilde{q}_1)\tilde{q}_1\int d\tilde{q}_2\, P(\tilde{q}_2\,|\,\tilde{q}_1)\tilde{q}_2 = \langle q_0^2\rangle\,. \quad (11)$$

This treatment can be easily extended to include as many measurements of $\hat{q}$ as we want, and a similar treatment can be used to analyze the results of the measurements of $\hat{p}$. Always the conclusion is the same: While it is possible to estimate the initial center positions of $P_0(q)$ and $P_0(p)$ with a linear function of the corresponding measurement results, no quadratic function of the results can estimate the initial widths of $P_0(q)$ and $P_0(p)$. Since no information about the widths of the probability densities of $\hat{q}$ and $\hat{p}$ is obtained, the process of repeated

QND measurements is equivalent to a measurement of the observables $\hat{q}$ and $\hat{p}$, and cannot be considered as a measurement of the wave function. The same is true for all processes of repeated Pauli's first-kind measurements of the arbitrary observable $\hat{q}\cos\theta + \hat{p}\sin\theta$.

The mathematical origin of this result is the dependence of each measurement on the specific results of the previous measurements. Physically, the measurement process modifies the wave function in accordance with the measurement result, i.e., the information which is extracted from the wave function. This modification is a direct result of the projection postulate. Therefore, regardless of the measurement strength, it is impossible to measure the wave function of a single system using repeated QND measurements.

We now illustrate these general considerations with two examples. The first example [7] is that of a series of photon-number QND measurements [8–11] performed on a single wave packet of light. Each time a measurement is performed, a probe, which is prepared in a squeezed state with a zero phase, $|\alpha_0, r\rangle_p$, where $|\alpha_0|$ is the initial excitation of the probe, and r is the squeezing parameter, is correlated to the signal in an optical Kerr medium. The correlation is described by the unitary operator $\hat{U} = \exp(i\mu\hat{n}_s\hat{n}_p)$, where $\hat{n}_s$ and $\hat{n}_p$ are the signal and probe photon number operators, respectively, and μ is the coupling strength [16]. Then, the second-quadrature amplitude of the probe, $\hat{a}_{2,p} \cong |\alpha_0|\hat{\varphi}_p$, where $\hat{\varphi}_p$ is the phase operator of the probe, is measured precisely by a homodyne detection. The measurement result, α_2, gives the inferred signal photon number, $\tilde{n} \cong \alpha_2/|\alpha_0|\mu$. The probability-amplitude operator $\hat{Y}(\hat{n},\tilde{n}) = {}_p\langle\tilde{n}|\hat{U}|\alpha_0, r\rangle_p$ corresponds to a Gaussian [17] transition probability, $X(n,\tilde{n}) = N[\tilde{n}, n, \Delta_m^2]$. The measurement error is $\Delta_m^2 = \langle\Delta\hat{a}_{2,p}{}^2\rangle/|\alpha_0|^2\mu^2$, where $\langle\Delta\hat{a}_{2,p}^2\rangle = e^{-2r}/4$ is the initial uncertainty of the second quadrature of the probe. The process of k repeated measurements is described by the total probability-amplitude operator, $\hat{Z}_k = \hat{Y}(\hat{n},\tilde{n}_k)\cdots\hat{Y}(\hat{n},\tilde{n}_2)\hat{Y}(\hat{n},\tilde{n}_1)$. Let us assume that the initial photon number distribution of the signal is a Gaussian, $P_0(n) = N[n, n_0, \Delta_0^2]$. Physically, the photon number has a discrete sub- or super-Poissonian distribution, where $n \geq 0$. If the initial signal excitation is large, i.e., $n_0 \gg 1$, this Gaussian approximation is valid. The final signal photon number distribution, $P_k(n) = N[n, n_0^k, \Delta_k^2]$, is then calculated according to Eq. (5),

$$n_0^k = \Delta_k^2\left(n_0/\Delta_0^2 + \sum_{i=1}^{k}\tilde{n}_i/\Delta_m^2\right), \qquad (12)$$

$$\Delta_k^2 = (1/\Delta_0^2 + k/\Delta_m^2)^{-1}. \qquad (13)$$

After each measurement the width of the photon number distribution decreases and its center shifts. The diffusion of the center after k measurements, n_0^k, is described statistically by $P_k(n_0^k) = N[n_0^k, n_0, (k/\Delta_m^2)\Delta_0^2\Delta_k^2]$. On average, the center is always at n_0. However, the probability of finding the center farther away from n_0 increases as the number of measurements increases. As long as the total strength of the measurements is small, $k/\Delta_m^2 \ll 1/\Delta_0^2$, the variance of n_0^k increases linearly with the number of measurements $(k/\Delta_m^2)\Delta_0^2\Delta_k^2 \cong Dk$. In this regime the movement of the center is a quantum Brownian motion with a constant diffusion coefficient $D = \Delta_0^4/\Delta_m^2$. As the photon number distribution narrows, the average step size of this Brownian motion decreases. The statistical variance of the center saturates, and equals the initial photon number uncertainty, Δ_0^2. At the same time, the wave function is reduced to a photon number eigenstate. The measured wave function, therefore, undergoes a quantum Brownian motion, which is saturated due to the continuous wave function collapse.

Using Eq. (1), the probability to obtain the measurement results $(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_k)$ is

$$P(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_k)\prod_{i=1}^{k} d\tilde{n}_i = \int_{-\infty}^{\infty} dn\, N[n, n_0, \Delta_0^2]\prod_{i=1}^{k} d\tilde{n}_i \times N[\tilde{n}_i, n, \Delta_m^2] = [P(\overline{n})\, d\overline{n}][P(S)\, dS]\, d\Omega_{k-1}, \qquad (14)$$

where $\overline{n} = \sum_{i=1}^{k}\tilde{n}_i/k$ and $\overline{\Delta n^2} = \sum_{i=1}^{k}(\tilde{n}_i - \overline{n})^2/(k-1)$ are the estimates for the center and the width of the initial photon number distribution, respectively [7], $S = [(k-1)/\Delta_m^2]\overline{\Delta n^2}$, and $d\Omega_{k-1}$ is a normalized infinitesimal element of the solid angle in $k-1$ dimensions, $\int d\Omega_{k-1} = 1$. The probability distribution of $\overline{n}$, $P(\overline{n}) = N[\overline{n}, n_0, \Delta_0^2 + \Delta_m^2/k]$, is centered at n_0. The variance of $\overline{n}$ decreases with an increased number of measurements, and as $k \to \infty$ this variance reaches its minimum value, Δ_0^2. The estimated center has the same probability error in both cases of an infinite number of repeated weak measurements and one precise measurement. The probability distribution of S is a chi-squared distribution [18], $P(S) = \chi^2[S, (k-1)]$, which is independent of Δ_0^2. Therefore, $\overline{\Delta n^2}$ is not a measure of the initial width of the wave function, Δ_0^2. Indeed, $\overline{\Delta n^2}$ is centered at Δ_m^2, with the variance $2\Delta_m^4/(k-1)$, i.e., $\overline{\Delta n^2}$ is a measure of the measurement error. We conclude that the statistics of the results of repeated weak QND measurements of the same observable, performed on a single system, contain no information about the initial width of its wave funciton, due to the exact coordination between the quantum Brownian motion and the continuous collapse of the wave function.

The second example shows that even when the collapse of the wave function to an eigenstate of the measured observable is prevented due to backaction noise imposed by measurements of the conjugate observable, it is still impossible to measure this wave function. This is the case of alternating QND measurements of the two quadrature amplitudes of a squeezed state, using a dual degenerate parametric amplification [13,14]. In the odd measurements, the result of a measurement of the second quadrature of the probe, $\hat{a}_{2,p}$, is used to infer the first quadrature of the signal, $\hat{a}_{1,s}$. Both the probe, which is in the vacuum state, and the signal have Gaussian distributions, and the previous model, of repeated photon number QND measurements, can be modified, to describe the distribution of $\hat{a}_{1,s}$ before the kth measurement, $P_{k-1}(\alpha) = N[\alpha, \alpha_0^{k-1}, \Delta_{k-1}^2]$. Taking into account the backaction noise due to the $(k-1)$th measurement of the second quadrature of the signal, $\hat{a}_{2,s}$, we obtain from Eqs. (12) and (13)

$$\alpha_0^{k-1} = (1/\Delta_{k-2}^2 + 1/\Delta_m^2)^{-1}(\alpha_0^{k-2}/\Delta_{k-2}^2 + \tilde{\alpha}_{k-1}/\Delta_m^2), \tag{15}$$

$$\Delta_{k-1}^2 = (1/\Delta_{k-2}^2 + 1/\Delta_m^2)^{-1} + \Delta_b^2, \tag{16}$$

where $\tilde{\alpha}_{k-1}$ is the result of the $(k-1)$th measurement of $\hat{a}_{1,s}$. $P_{k-1}(\alpha)$ determines the conditional probability to obtain $\tilde{\alpha}_k$ in the kth measurement, $P(\tilde{\alpha}_k \mid \tilde{\alpha}_{k-1}, \ldots, \tilde{\alpha}_1)$, according to Eqs. (1)–(4). This allows us to calculate the second-order moment,

$$\langle \tilde{\alpha}_k^2 \rangle = \int d\tilde{\alpha}_1 \, P(\tilde{\alpha}_1) \int d\tilde{\alpha}_2 \, P(\tilde{\alpha}_2 \mid \tilde{\alpha}_1) \cdots \int d\tilde{\alpha}_k \, P(\tilde{\alpha}_1 \mid \tilde{\alpha}_{i-1}, \ldots, \tilde{\alpha}_1) \tilde{\alpha}_k^2 = \alpha_0^2 + \Delta_0^2 + \Delta_m^2 + (k-1)\Delta_b^2, \tag{17}$$

and the correlation, for all $j \geq k+1$,

$$\langle \tilde{\alpha}_k \tilde{\alpha}_j \rangle = \int d\tilde{\alpha}_1 \, P(\tilde{\alpha}_1) \cdots \int d\tilde{\alpha}_k \, P(\tilde{\alpha}_k \mid \tilde{\alpha}_{k-1}, \ldots, \tilde{\alpha}_1) \tilde{\alpha}_k \cdots \int d\tilde{\alpha}_j \, P(\tilde{\alpha}_j \mid \tilde{\alpha}_{j-1}, \ldots, \tilde{\alpha}_1) \tilde{\alpha}_j = \alpha_0^2 + \Delta_0^2 + (k-1)\Delta_b^2, \tag{18}$$

where $P_0(\alpha) = N[\alpha, \alpha_0, \Delta_0^2]$ is the initial distribution of $\hat{a}_{1,s}$. From Eqs. (17) and (18) we see that the information about Δ_0^2 is always "screened" by α_0^2, and therefore is impossible to obtain. The same treatment can be repeated using the measurement results of $\hat{a}_{2,s}$. The wave function is prevented from collapsing to an eigenstate of $\hat{a}_{1,s}$ (or $\hat{a}_{2,s}$), but the narrowing and widening of the wave function due to the alternating measurements of $\hat{a}_{1,s}$ and $\hat{a}_{2,s}$ would, eventually, balance, to keep the width of the wave function the same each time $\hat{a}_{1,s}$ (or $\hat{a}_{2,s}$) is measured, i.e., $\Delta_{k-1}^2 = \Delta_k^2$. In this limit, the wave function undergoes a process of free diffusion, preserving its noise distribution. This final noise distribution of the wave function is determined solely by the relative strengths of the $\hat{a}_{1,s}$ and $\hat{a}_{2,s}$ measurements. If these measurements have equal strengths, for instance, the noise distribution of the wave function would be that of a coherent state. As in the previous example, the coordination between the shifts of the center and the changes in the width, which are caused to the wave function by the repeated measurements, inhibit the measurement of this wave function.

To conclude, we have shown that the wave function of a single quantum system cannot be measured by a series of weak QND measurements without an *a priori* knowledge of the wave function. During the measurement process, the wave function undergoes quantum Brownian motion and continuous collapse. Due to this physical mechanism, which originates in the projection postulate, the statistics of the measurement results contain no information about the initial width of the wave function.

It is our pleasure to acknowledge helpful discussions with N. Imoto. One of us (O. A.) also acknowledges useful discussions with L. Vaidman.

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[17] In this Letter $N[x, x_0, \sigma^2]$ is defined to be a normal distribution of the variable x, which is centered at x_0 with the variance σ^2, $N[x, x_0, \sigma^2] = (2\pi\sigma^2)^{-1/2} \exp[-(x - x_0)^2/2\sigma^2]$.

[18] In this Letter $\chi^2[x, \nu]$ is defined to be a chi-squared distribution of the variable x, which is centered at ν with the variance 2ν, $\chi^2[x, \nu] = 2^{\nu/2}\Gamma(\nu/2)^{-1} x^{(\nu-2)/2} \exp(-x/2)$.