Quantum Zeno effect and the impossibility of determining the quantum state of a single system

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The quantum Zeno effect of a single system is the effect of a series of measurements on the unitary time evolution of the system and on the ability to monitor this time evolution using the measurement results. This effect is shown to be equivalent to that of the indetermination of the quantum state of a single system [Phys. Rev. Lett. **74**, 4106 (1995)], in which one considers the statistics of the results of a series of measurements performed on a single system, with no time evolution in between successive measurements. [S1050-2947(97)51204-2]

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The quantum Zeno effect, which was introduced by Misra and Sudarshan ([1], and see also [2,3]), is the effect of a series of quantum measurements, which induce reduction, on the unitary quantum time evolution. This effect is manifested in two types of (thought) experiments. In the first type, the same series of measurements is performed on each system in an ensemble of systems, all of which are prepared initially in the same pure state. This type of experiment corresponds to the quantum Zeno effect of an ensemble, in which one considers the changes in the unitary time evolution of an ensemble of identical systems due to a series of measurements. These changes were shown to be indistinguishable from those due to dephasing (see, e.g., [4–7]), i.e., the predictions of quantum mechanics regarding the changes in the time evolution of the ensemble due to the series of measurements are independent of the measurement results. Therefore it was also suggested that the quantum Zeno effect is a dynamical effect rather than a quantum measurement effect (see, e.g., [8–10]).

In this Rapid Communication we consider the second type of (thought) experiments, in which the same series of measurements is performed on each system in an ensemble of systems, all of which are initially in the same pure state, where after each measurement the measured system is retained only if the measurement result equals the result that is obtained by the first system to be measured; otherwise the system is discarded. After each measurement, one is left with the subensemble of systems, which is defined not only by the initial pure state but also by the series of results obtained in the series of measurements. This type of experiment corresponds to the quantum Zeno effect of a single system, in which one considers the following two questions: How would the unitary time evolution of a single quantum system change due to a series of quantum measurements? What information about the unitary time evolution of the single system could be obtained from the series of measurement results? We show that, in the frame of reference that evolves in time with the system, these questions regard the indetermination of the unknown quantum state of a single system [11–13]: How would the initial quantum state of a single system change due to a series of measurements? What information about the initial quantum state of the single system could be obtained from the series of measurement results? It was already suggested that unlike the ensemble effect, the quantum Zeno effect of a single system cannot be thought of as a dephasing effect (see, e.g., [14,15]). We show that the quantum Zeno effect of a single system and the indetermination of the quantum state of a single system are two different descriptions, in the Schrödinger and the Heisenberg pictures, respectively, of the same phenomenon. Since the indetermination of the quantum state of a single system is due to the reduction induced by the measurement process, we suggest that the quantum Zeno effect of a single system is more than a dephasing effect, i.e., it is truly a measurement effect.

The following example of a two-level atom and a single-photon mode in a cavity illustrates the relation between the quantum Zeno effect of a single system and the indetermination of the unknown state of this system. The two-level atom is described by its excited state $|e\rangle$, its ground state $|g\rangle$, and the energy separation between them, $\hbar\omega$. The single-photon mode is described by the single-photon state $|1\rangle$, the vacuum state $|0\rangle$, and the photon energy $\hbar\nu$. On resonance, $\omega=\nu$, with the rotating-wave approximation, where the energy of the state $|e\rangle|0\rangle$ is taken as the zero-energy point, the Jaynes-Cummings Hamiltonian of the two-level atom and the single-photon mode in the cavity is $\hat{H}=\hbar g\hat{\sigma}_x$, where $\hbar g$ is the strength of the interaction between the atom and the photon mode, and where

$$\hat{I}=|e\rangle|0\rangle\langle e|\langle 0|+|g\rangle|1\rangle\langle g|\langle 1|,$$

$$\hat{\sigma}_x=|e\rangle|0\rangle\langle g|\langle 1|+|g\rangle|1\rangle\langle e|\langle 0|,$$

$$\hat{\sigma}_y=-i(|e\rangle|0\rangle\langle g|\langle 1|-|g\rangle|1\rangle\langle e|\langle 0|),$$

$$\hat{\sigma}_z=|e\rangle|0\rangle\langle e|\langle 0|-|g\rangle|1\rangle\langle g|\langle 1| \tag{1}$$

are the identity operator and the fictitious spin components in the space $\{|e\rangle|0\rangle,|g\rangle|1\rangle\}$, respectively. At $t=0$ the atom-photon system is in the pure state

$$|\psi(0)\rangle=\cos(\theta/2)\exp(-i\phi/2)|e\rangle|0\rangle + \sin(\theta/2)\exp(i\phi/2)|g\rangle|1\rangle. \tag{2}$$

The free time evolution of the atom-photon system is determined by the unitary operator $\hat{U}(t)=\cos(gt)\hat{I}-i\sin(gt)\hat{\sigma}_x$, where $|\psi(t)\rangle=\hat{U}(t)|\psi(0)\rangle$. One manifestation of this deterministic unitary time evolution is the oscillation of the photon number in the cavity, with the Rabi frequency $\Omega_R=2g$,

$$\langle \psi(t)|\hat{n}_c|\psi(t)\rangle = \sin^2(\theta/2)\cos^2(gt) + \sin^2(gt)\cos^2(\theta/2) + \sin(\phi)\sin(\theta)\sin(2gt)/2, \quad (3)$$

where $\hat{n}_c = |g\rangle|1\rangle\langle g|\langle 1|$. As is well known, a record of the oscillations of the photon number in the cavity, if available, would indicate the strength of the atom-photon interaction. We would like to emphasize that a record of $\langle \psi(t)|\hat{n}_c|\psi(t)\rangle$ would also indicate the initial state of the atom-photon system: $\sin(\theta)$ would determine $\theta \in [0,\pi]$ and $\sin(\phi)$ would determine $\phi \in [0, 2\pi]$ up to a twofold degeneracy, ϕ and $\pi - \phi$. Therefore, one could determine the initial atom-photon state up to a twofold degeneracy.

In the fictitious spin space, the Hamiltonian of the atom-photon system, $\hat{H} = -\vec{\Omega}_R \cdot (\hbar \hat{\vec{\sigma}}/2)$ with $\vec{\Omega}_R = -2g\vec{e}_x$, where $\vec{e}_x$ is a unit vector in the $\vec{x}$ direction, corresponds to the precession of the fictitious spin of the system in the reference frame $\{\vec{x},\vec{y},\vec{z}\}$, $d\hat{\vec{\sigma}}(t)/dt = -\vec{\Omega}_R \times \hat{\vec{\sigma}}(t)$, where $\hat{\vec{\sigma}}(0) = 2\hat{\rho}(0) - \hat{I}$ and where $\hat{\rho}(0) = |\psi(0)\rangle\langle\psi(0)|$ is the initial density operator. As the fictitious spin precesses, its projection onto the $\vec{z}$ axis oscillates. These are the Rabi oscillations of the energy of the system between the atom and the photon mode. In the frame of reference that rotates with the fictitious spin, $\{\vec{x}',\vec{y}',\vec{z}'\}$, where $\vec{x}' = \vec{x}$, $\vec{y}' = \vec{y}\cos(\Omega_R t) - \vec{z}\sin(\Omega_R t)$ and $\vec{z}' = \vec{z}\cos(\Omega_R t) + \vec{y}\sin(\Omega_R t)$, the spin always points in its initial direction, but the $\vec{y}$ and $\vec{z}$ axes precess in time. The time evolution of the photon number in the cavity is now a record of the projection of the fixed spin onto the time precessing axis $\vec{z} = \vec{z}'\cos(\Omega_R t) - \vec{y}'\sin(\Omega_R t)$. In order to determine the initial direction of the fictitious spin one needs to know its projections along the $\vec{x}$, $\vec{y}$, and $\vec{z}$ axes; i.e., one needs to measure $\hat{\sigma}_x$, $\hat{\sigma}_y$, and $\hat{\sigma}_z$. Indeed, the process of free precession in time interrupted by repeated measurements of $\hat{\sigma}_z$ appears in the rotating frame of reference as a series of measurements of the time-varying observable $\hat{\sigma}_z = \hat{\sigma}_z'\cos(\Omega_R t) - \hat{\sigma}_y'\sin(\Omega_R t)$. These measurements provide information about the projection of the fictitious spin in the $\{\vec{y},\vec{z}\}$ plane only, from which one could determine the initial direction of the spin up to a twofold degeneracy due to mirror symmetry about the $\{\vec{y},\vec{z}\}$ plane.

Assume now that one was trying to obtain a record of the unitary time evolution of the atom-photon system, from which the initial state of the system may be deduced, using a series of back-action evading (BAE) measurements of the photon number in the cavity [16]. Each time a measurement is performed, the atom-photon state is correlated in an optical Kerr medium in the cavity to a squeezed probe state $|\alpha,r\rangle_p$, with the excitation $|\alpha|^2$ and the squeezing parameter r. This process is described by the unitary operator $\hat{U}(\hat{n}_c) = \exp(i\mu\hat{n}_c\hat{n}_p)$, where $\hat{n}_c$ and $\hat{n}_p$ are the photon-number operators of the cavity mode and the probe, respectively, and μ is the coupling strength. The second-quadrature amplitude of the probe, $\hat{a}_{2,p}$, is measured precisely; and if the initial phase of the probe is zero, and the coupling is weak, $\mu \ll 1$, then the measurement result α_2 gives the inferred photon number $\tilde{n} \cong \alpha_2/(|\alpha|\mu)$ with the inference error $\Delta^2 = e^{-2r}/(2|\alpha|\mu)^2$. The probability-amplitude operator [7] that describes this measurement process is

$$\hat{Y}(\hat{n}_c,\tilde{n}) = {}_p\langle \tilde{n}|\hat{U}(\hat{n}_c)|\alpha,r\rangle_p = Y_{11}(\tilde{n})|e\rangle|0\rangle\langle e|\langle 0| + Y_{22}(\tilde{n})|g\rangle|1\rangle\langle g|\langle 1|,$$

$$Y_{11}(\tilde{n}) = \exp[-\tilde{n}^2/(4\Delta^2)]/\sqrt[4]{2\pi\Delta^2},$$

$$Y_{22}(\tilde{n}) = \exp[-(\tilde{n}-1)^2/(4\Delta^2)]/\sqrt[4]{2\pi\Delta^2}, \quad (4)$$

where it is assumed, without loss of generality, that $\exp(\pm i\alpha^2\mu) = 1$. Note that this is a general form for all photon-number BAE measurements, for which $\tilde{n}$ and Δ are the measurement result and error, respectively. The state of the atom-photon system after the measurement is $\hat{\rho}_a(\tilde{n}) = P(\tilde{n})^{-1}\hat{Y}(\hat{n}_c,\tilde{n})\hat{\rho}_b\hat{Y}^\dagger(\hat{n}_c,\tilde{n})$, where

$$\hat{\rho}_b = \rho_{11}|e\rangle|0\rangle\langle e|\langle 0| + \rho_{12}|e\rangle|0\rangle\langle g|\langle 1| + \rho_{21}|g\rangle|1\rangle\langle e|\langle 0| + \rho_{22}|g\rangle|1\rangle\langle g|\langle 1| \quad (5)$$

is the atom-photon state before the measurement and $P(\tilde{n}) = |Y_{11}(\tilde{n})|^2\rho_{11} + |Y_{22}(\tilde{n})|^2\rho_{22}$ is the probability of obtaining the measurement result $\tilde{n}$.

The effect of the measurement on an ensemble of systems, which is independent of the measurement results, is to reduce the coherences, i.e., the off-diagonal elements, of the ensemble density operator and therefore it appears to be similar to the effect of dephasing,

$$\hat{\rho}_a = \rho_{11}|e\rangle|0\rangle\langle e|\langle 0| + \rho_{22}|g\rangle|1\rangle\langle g|\langle 1| + \exp[-1/(8\Delta^2)] \times (\rho_{12}|e\rangle|0\rangle\langle g|\langle 1| + \rho_{21}|g\rangle|1\rangle\langle e|\langle 0|). \quad (6)$$

This is why quantum dephasing is considered to be a consequence of a measurement process, where the corresponding measurement result cannot be recorded. Indeed, the quantum Zeno effect of an ensemble, the effect of a series of measurements on the time evolution of an ensemble of systems, was shown to be indistinguishable from the effect of dephasing (see, e.g., [4–7]). So is the case in our example, where the series of photon-number measurements introduce dephasing of the unitary photon-number oscillations. When the measurements are imprecise, $\Delta > 1$, their disturbance of the coherence between the atom and the photon mode is small and the photon-number oscillations are underdamped [Fig. 1(a)], and when the measurements are precise, $\Delta < 1$, the photon-number oscillations are overdamped [Fig. 2(a)]. Yet, the time dependence of the ensemble average of the results of the photon-number measurements, i.e., the time evolution of the expectation value of the cavity photon number, would allow determination of the initial state of the identical atom-photon systems, even though it includes the damping that is induced by the series of measurements. Also, the unitary time evolution of the photon number, i.e., the Rabi frequency of the photon-number oscillations, can be determined from the results of a series of photon-number measurements performed on an ensemble of atom-photon systems.

The results of a series of photon-number BAE measurements performed on a single atom-photon system, however, can be used to determine neither the unitary time evolution

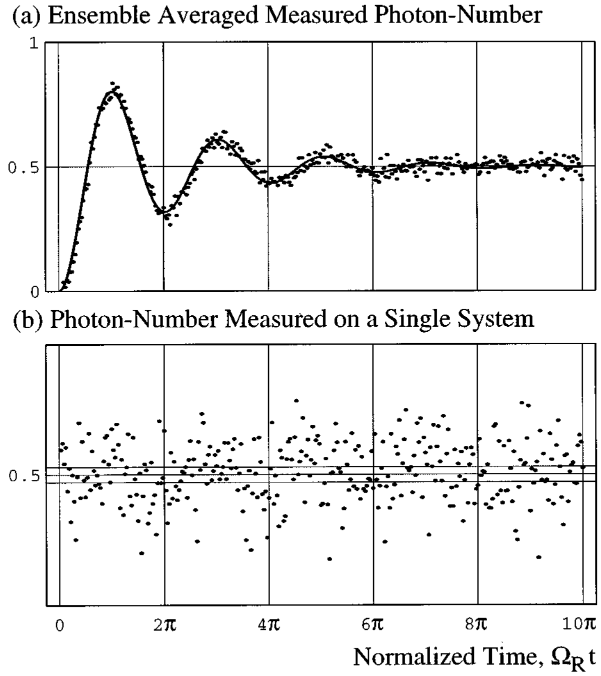

FIG. 1. A series of imprecise photon-number measurements, with the error $\Delta=2$ at a rate of $32\,\Omega_R/\pi$, performed on an atom-photon system with $|\psi(0)\rangle=|e\rangle|0\rangle$. (a) The averages over the ensemble measurement results determine the underdamped oscillations of the photon number in the cavity; (b) the results obtained from a single system cannot estimate these underdamped oscillations.

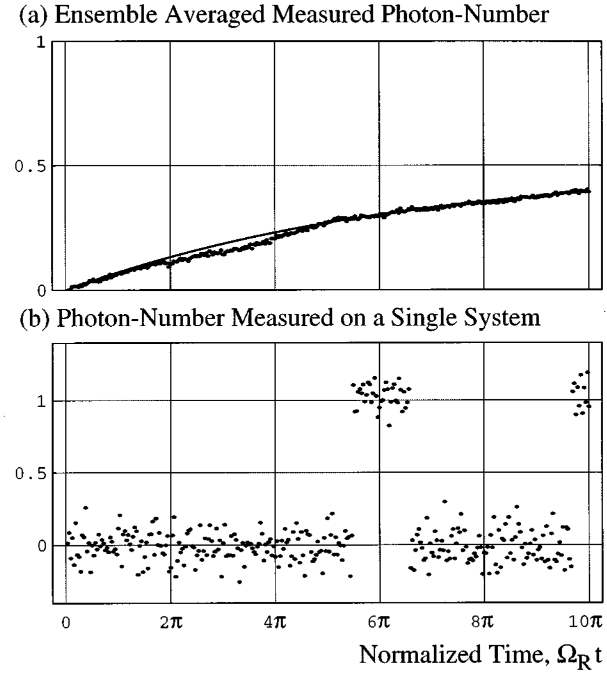

FIG. 2. A series of precise photon-number measurements, with the error $\Delta=0.1$ at a rate of $32\,\Omega_R/\pi$, performed on an atom-photon system with $|\psi(0)\rangle=|e\rangle|0\rangle$. (a) The averages over the ensembe measurement results determine the overdamped oscillations of the photon number in the cavity; (b) the quantum jumps exhibited by the results that are obtained from a single system cannot estimate these underdamped oscillations.

of the photon number in the cavity, nor the initial state of the single system [Figs. 1(b) and 2(b)]. This is because, in the process of the quantum measurement, the state of the single system is changed according to the measurement result. Each of the results of a series of measurements performed on a single system depends statistically on all of the previous measurement results, regardless of the precision of the measurements. Due to their statistical dependence, these results cannot determine the initial quantum state of the single system fully [11,12], and in our example they also cannot determine the unitary time evolution of the single system.

In this example, knowledge of the initial state of the single atom-photon system corresponds to knowledge of the unitary time evolution of this system. A series of measurements, which is designed to obtain the unitary time evolution of the single system, would have the same effect on the system as a series of measurements that is designed to obtain the initial quantum state of this system in the absence of unitary time evolution.

Recently we showed that the quantum state that describes a single system cannot be determined from the results of a series of measurements performed on the single system ([11,12] and see also [13]). Let us now show that this effect of the indetermination of the unknown quantum state of a single system is equivalent to the quantum Zeno effect of a single system. Consider a series of n measurements of the observable $\hat{q}$ performed on a single quantum system during its unitary time evolution in the time interval $t\in[0,T]$. The initial state of the system is described by the density operator $\hat{\rho}_0$, and the deterministic time evolution of the system in between the $(k-1)$th and the kth measurements at $t_{k-1}=(k-1)T/n$ and $t_k=kT/n$, respectively, is described by the unitary operator $\hat{U}_k$, $\hat{\rho}_k=\hat{U}_k\hat{\rho}_{k-1}\hat{U}_k^\dagger$. The kth measurement process at $t_k=kT/n$, i.e., the preparation of the kth probe in the pure state $|\phi\rangle_{p,k}$, the interaction of this probe with the measured system $\hat{U}_M(\hat{q})$, and the result of the measurement $\tilde{q}_k$, which corresponds to the state of the probe after the measurement $|\tilde{q}_k\rangle_{p,k}$, is described by the probability-amplitude operator $\hat{Y}_k\equiv\hat{Y}(\hat{q},\tilde{q}_k)={}_{p,k}\langle\tilde{q}_k|\hat{U}_M(\hat{q})|\phi\rangle_{p,k}$. The state of the system after the kth measurement is $\hat{\rho}_{a,k}=P(\tilde{q}_k)^{-1}\hat{Y}_k\hat{\rho}_{b,k}\hat{Y}_k^\dagger$, where $\hat{\rho}_{b,k}$ is the state of the system before this measurement; and $P(\tilde{q}_k)=\mathrm{Tr}_s[\hat{Y}_k\hat{\rho}_{b,k}\hat{Y}_k^\dagger]$, with the trace over the operators of the measured system, is the probability to obtain the measurement result $\tilde{q}_k$. The density operator, which describes the single system at $t=T$, after the nth measurement is

$$\hat{\rho}_S=P_S(\tilde{q}_1,\dots,\tilde{q}_n)^{-1}\hat{Y}_n\hat{U}_n\cdots\hat{Y}_1\hat{U}_1\hat{\rho}_0\hat{U}_1^\dagger\hat{Y}_1^\dagger\cdots\hat{U}_n^\dagger\hat{Y}_n^\dagger. \tag{7}$$

The probability density of obtaining the series of measurement results $(\tilde{q}_1,\dots,\tilde{q}_n)$ is

$$\begin{aligned}P_S(\tilde{q}_1,\dots,\tilde{q}_n)&=\mathrm{Tr}_s[\hat{Y}_n\hat{U}_n\cdots\hat{Y}_1\hat{U}_1\hat{\rho}_0\hat{U}_1^\dagger\hat{Y}_1^\dagger\cdots\hat{U}_n^\dagger\hat{Y}_n^\dagger]\\&=\mathrm{Tr}_s[\hat{Z}_n\cdots\hat{Z}_1\hat{\rho}_0\hat{Z}_1^\dagger\cdots\hat{Z}_n^\dagger]\\&=P_H(\tilde{q}_1,\dots,\tilde{q}_n),\end{aligned} \tag{8}$$

where we used the unitarity of the time evolution operators, $\hat{U}_k\hat{U}_k^\dagger=\hat{I}$, with $\hat{I}$ being the identity operator and where $\hat{Z}_k \equiv \hat{Z}(\hat{q}_k, \tilde{q}_k) = \hat{U}_1^\dagger \cdots \hat{U}_k^\dagger \hat{Y}_k \hat{U}_k \cdots \hat{U}_1$. Note that $P_H(\tilde{q}_1, \ldots, \tilde{q}_n)$ describes the probability density to obtain the series of results $(\tilde{q}_1, \ldots, \tilde{q}_n)$ in the series of measurements $(\hat{Z}_1, \ldots, \hat{Z}_n)$ of the single system, with no time evolution in between successive measurements. The state of the system after this series of measurements is described by $\hat{\rho}_H = P_H(\tilde{q}_1, \ldots, \tilde{q}_n)^{-1} \hat{Z}_n \cdots \hat{Z}_1 \hat{\rho}_0 \hat{Z}_1^\dagger \cdots \hat{Z}_n^\dagger$.

Since the statistics $P_S(\tilde{q}_1, \ldots, \tilde{q}_n)$ and $P_H(\tilde{q}_1, \ldots, \tilde{q}_n)$ are equal, the physical processes that they describe are equivalent. While $\hat{\rho}_S$ is the density operator of the system at $t=T$ in the Schrödinger picture, where the time evolution is attributed to the state of the system, $\hat{\rho}_H$ is the density operator of this system in the Heisenberg picture, where the time evolution is attributed to the observables associated with the system and therefore also to the probability-amplitude operators. In fact, $\hat{\rho}_H$ could be viewed as the state of the system at $t=T$ in the reference frame that evolves in time with the system. Indeed, in the specific case of BAE measurements, where $[\hat{Y}_k, \hat{q}] = [\hat{U}_M(\hat{q}), \hat{q}] = 0$, the probability-amplitude operators in the Heisenberg picture describe successive measurements of the time-evolving observable $\hat{q}_k = \hat{U}_1^\dagger \cdots \hat{U}_k^\dagger \hat{q} \hat{U}_k \cdots \hat{U}_1$. For example, the case of repeated photon-number BAE measurements of a two-level atom and a single-photon mode in a cavity can be viewed in the space of the fictitious spin associated with the atom-photon system using the Schrödinger picture, in which the photon-number measurements correspond to measurements of $\hat{\sigma}_z$, the $\vec{z}$ component of the precessing spin $\hat{\vec{\sigma}}(t)$. In the Heisenberg picture, i.e., in the frame of reference that rotates with the spin, the photon-number measurements correspond to repeated measurements of the precessing observable $\hat{\sigma}_z = \hat{\sigma}_z' \cos(\Omega_R t) - \hat{\sigma}_y' \sin(\Omega_R t)$.

The Schrödinger and the Heisenberg pictures describe equivalent physical phenomena. In the Schrödinger picture $\hat{\rho}_S$ describes the effect of a series of measurements of the same observable on the free time evolution of a single system, and $P_S(\tilde{q}_1, \ldots, \tilde{q}_n)$ describes the information about the free time evolution of the system that is contained in the measurement results. The Schrödinger picture, therefore, describes the quantum Zeno effect of a single system. In the Heisenberg picture $\hat{\rho}_H$ describes the ''inverse'' quantum Zeno effect of a single system [17,18], i.e., the stochastic ''time evolution'' of a single quantum system due to a series of measurements of time-varying observables; and $P_H(\tilde{q}_1, \ldots, \tilde{q}_n)$ describes the indetermination of the unknown quantum state of a single system. The quantum Zeno effect of a single system is equivalent to this effect of the indetermination of the unknown quantum state of a single system.

Presenting the quantum Zeno effect of a single system in the Heisenberg picture, it is obvious that the only changes in the state of the system arise from the measurement process, due to the reduction that is induced by the measurement. These changes prohibit the determination of the quantum state of the single system using the measurement results. The quantum Zeno effect of a single system can be thought of as a ''screening'' effect: The series of measurements change the unitary time evolution of the single system in such a way that the initial state of the system cannot be determined from the measurement results.

We have shown that the quantum Zeno effect of a single system and the indetermination of the unknown quantum state of a single system are two descriptions, in the Schrödinger and the Heisenberg pictures, respectively, of the same phenomenon: the effect of a series of measurements on the state of the single system, as a consequence of the reduction that is induced by the measurements. In the Heisenberg picture the series of measurement results cannot give full information about the initial quantum state of the system, and in the Schrödinger picture these results cannot give full information about the unitary time evolution of the single system. The quantum Zeno effect of a single system is, therefore, more than a dephasing effect, it is a quantum measurement effect.

[1] B. Misra and E. C. G. Sudarshan, J. Math. Phys. (N.Y.) **18**, 756 (1977).
[2] L. A. Khalfin, Pis'ma Zh. Eksp. Teor. Fiz. **8**, 106 (1968) [JETP Lett. **8**, 65 (1968)].
[3] L. Fonda *et al.*, Nuovo Cimento A **15**, 689 (1973).
[4] G. J. Milburn, J. Opt. Soc. Am. B **5**, 1317 (1988).
[5] A. Peres and A. Ron, Phys. Rev. A **42**, 5720 (1990).
[6] T. F. Jordan *et al.*, Phys. Rev. A **44**, 3340 (1991).
[7] V. B. Braginsky and F. Ya. Khalili, *Quantum Measurement* (Cambridge University Press, Cambridge, MA, 1992).
[8] V. Frerichs and A. Schenzle, Phys. Rev. A **44**, 1962 (1991).
[9] S. Pascazio *et al.*, Phys. Lett. A **179**, 155 (1993).
[10] S. Pascazio and M. Namiki, Phys. Rev. A **50**, 4582 (1994).
[11] O. Alter and Y. Yamamoto, Phys. Rev. Lett. **74**, 4106 (1995).
[12] O. Alter and Y. Yamamoto, in *Quantum Physics, Chaos Theory and Cosmology*, edited by M. Namiki *et al.*, (AIP, New York, 1996), pp. 151–172.
[13] G. M. D'Ariano and H. P. Yuen, Phys. Rev. Lett. **76**, 2832 (1996).
[14] T. P. Spiller, Phys. Lett. A **192**, 163 (1994).
[15] W. L. Power and P. L. Knight, Phys. Rev. A **53**, 1052 (1996).
[16] K. Kitagawa *et al.*, Phys. Rev. A **35**, 5270 (1987).
[17] Y. Aharonov and M. Vardi, Phys. Rev. D **21**, 2235 (1980).
[18] T. P. Altenmüller and A. Schenzle, Phys. Rev. A **48**, 70 (1993).