Reply to ''Comment on 'Protective measurement of the wave function of a single squeezed harmonic-oscillator state' ''

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The adiabatic ''protective measurement'' suggested by Aharonov, Anandan, and Vaidman [Phys. Lett. A **178**, 38 (1993)] is a measurement of a discrete energy eigenstate, in which the entanglement between the measured state and the quantum probe is avoided approximately when they interact adiabatically, and the measured state is left approximately unchanged (up to a phase factor) for all measured observables. This measurement scheme, however, does not overcome the reduction of the measured state, because this state is only approximately, and not exactly, disentangled from the quantum probe. Our recent analysis of a measurement scheme of a squeezed harmonic-oscilllator state [Phys. Rev. A **53**, R2911 (1996)], in which the entanglement between the measured state and the probe is avoided exactly, and the measured state is left exactly unchanged (up to a phase factor) for all measured observables, suggests that in order to measure the quantum state of a single system without changing this state, full *a priori* information about this state is required. [S1050-2947(97)08206-1]

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The indirect measurement of a quantum system, i.e., a quantum signal, is composed of three stages: preparation of a quantum probe, interaction of the probe with the signal, and measurement of an observable of the probe, which induces collapse (or reduction), of the probe. Usually, the probe and the signal are entangled after their interaction and the collapse of the probe changes the quantum state of the signal. Aharonov, Anandan, and Vaidman [1,2] recently suggested that the entanglement of the signal and the probe can be approximately avoided when the signal is known to be in a discrete energy eigenstate, which they call a ''protected state,'' by having the signal interact adiabatically with the probe. According to the adiabatic approximation, if the interaction of the signal with the probe is turned on and then turned off sufficiently slowly, then the signal and the probe are left approximately disentangled after this interaction, where the signal is approximately back in its initial state (up to a phase factor). In general, when the probe is prepared initially in any state other than an energy eigenstate, the interaction with the signal changes the state of the probe and a subsequent measurement of the probe yields information about the signal, leaving the state of the signal approximately unaffected. The adiabatic ''protective measurement'' seems to allow a series of measurements of all of the observables that are associated with the signal to be performed on the signal without changing it, even if these observables do not commute with each other. Therefore, the adiabatic protective measurement seems to allow a determination of the quantum state of a single system without full *a priori* knowledge of this state. Yet, approximate disentanglement of the signal and the probe is not sufficient to *protect* the state of the signal from reduction. For that purpose, exact disentanglement of the signal and the probe is necessary. The reduction was shown to prohibit the determination of the quantum state of a single system [3,4] (see also [5]). Therefore, the adiabatic approximation, which approximates an actual entanglement of states as disentanglement, is not valid in the analysis of a quantum measurement process of a single system.

For example, consider a measurement of the generalized position $\hat{s}_1$ of a quantum harmonic oscillator, which is initially in the number state $|n\rangle_s$. The number states are energy eigenstates of the harmonic oscillator, and therefore any number state is a protected state, as suggested by Aharonov and Vaidman in their Comment [6]. The free Hamiltonian of the harmonic oscillator is $\hat{H}_0=\hbar\omega(\hat{s}_1^2+\hat{s}_2^2)$, where $\hat{s}_2$ is the generalized momentum of the oscillator. The interaction of the signal with the probe, in which the generalized position of the signal $\hat{s}_1$ is coupled to the generalized momentum of the probe $\hat{p}_2$, is described by the Hamiltonian $\hat{V}(t)=2\hbar\kappa(t)\hat{s}_1\hat{p}_1$. The time evolution of the measured harmonic oscillator state is governed by the Hamiltonian $\hat{H}(t)=\hat{H}_0+\hat{V}(t)$. Using normal ordering of the unitary time evolution operator, it can be shown [7,8] that when the signal is initially in a coherent state $|\alpha\rangle_s$ and the probe is in a generalized position eigenstate $|\beta_1\rangle_p$, where $\hat{p}_1|\beta_1\rangle_p=\beta_1|\beta_1\rangle_p$, then after their interaction, the signal and probe are disentangled, where the signal is left in the coherent state $|\exp(-i\omega t)[\alpha+\delta(t)]\rangle_s$ and the probe is left in its initial state $|\beta_1\rangle_p$ (up to a phase factor):

$$\hat{U}(t)|\alpha\rangle_s|\beta_1\rangle_p=\exp[i\phi(t)]|\exp(-i\omega t)[\alpha+\delta(t)]\rangle_s|\beta_1\rangle_p,$$

$$\delta(t)=-i\beta_1\int_0^t\kappa(t')\exp(i\omega t')dt',$$

$$\phi(t)=\int_0^t|\delta(t')|^2\frac{d\arg[\delta(t')]}{dt'}dt'+\frac{1}{2i}[\alpha^*\delta(t)-\alpha\delta^*(t)]. \tag{1}$$

Using this result, it can be shown (after some math) that when the signal is initially in the number state $|n\rangle_s=\int(d^2\alpha/\pi)_s\langle\alpha|n\rangle_s|\alpha\rangle_s$ and the probe is in the generalized position eigenstate $|\beta_1\rangle_p$, then again the signal and probe are disentangled after their interaction, where the probe is left in its initial state $|\beta_1\rangle_p$:

$$\hat{U}(t)|n\rangle_s|\beta_1\rangle_p=\exp[-|\delta(t)|^2/2]\Bigg\{\sum_{k=0}^{n-1}\sqrt{k!/n!}\times[-\exp(i\omega t)\delta^*(t)]^{n-k}L_k^{n-k}[|\delta(t)|^2]+\sum_{k=n}^{\infty}\sqrt{n!/k!}[\exp(-i\omega t)\delta(t)]^{k-n}\times L_n^{k-n}[|\delta(t)|^2]\Bigg\}|k\rangle_s|\beta_1\rangle_p, \tag{2}$$

where $L_n^k[|\delta(t)|^2]$ is the generalized Laguerre polynomial of the variable $|\delta(t)|^2$. Note that the final state of the signal depends on β_1, the initial generalized position of the probe.

Now, according to the adiabatic approximation [9], if the turn-on and turn-off of the interaction $\hat{V}(t)$ are sufficiently slow, the probability amplitude $a_k(t)$ for the transition of the signal from its initial number state $|n\rangle_s$ to any other number state $|k\rangle_s$, where $k\neq n$ is small, is

$$|a_k(t)|\approx\left|\int_0^t\frac{1}{\hbar\omega(k-n)}\,{}_s\!\left\langle k\left|\frac{\partial\hat{V}(t')}{\partial t'}\right|n\right\rangle_s\times\exp[i\omega(k-n)t']dt'\right|\ll 1 \tag{3}$$

and therefore can be neglected, i.e., $a_k(t)\approx 0$. This leads to the approximation

$$\int_0^t\frac{\beta_1}{\omega}\frac{d\kappa(t')}{dt'}\exp(i\omega t')dt'\approx 0. \tag{4}$$

With this approximation, after the interaction is turned off at $t=T$, where $\kappa(0)=\kappa(T)=0$, evaluation of $\delta(T)$ of Eq. (1) using integration by parts gives

$$\delta(T)=-\frac{\beta_1}{\omega}[\kappa(T)\exp(i\omega T)-\kappa(0)]+\int_0^T\left[\frac{\beta_1}{\omega}\frac{d\kappa(t)}{dt}\exp(i\omega t)\right]dt\approx 0. \tag{5}$$

Substituting this in Eq. (2), one obtains

$$\hat{U}(T)|n\rangle_s|\beta_1\rangle_p\approx|n\rangle_s|\beta_1\rangle_p. \tag{6}$$

The exact solution to the time evolution problem, i.e., Eq. (2), shows that the initial number state of the signal evolves to a superposition of number states, which depends on the initial generalized position of the probe β_1. The approximated solution, i.e., Eq. (6), suggests that the state of the signal has not been changed at all. Now consider the case in which the probe is initially in a superposition of generalized position eigenstates. Since a measurement of the generalized momentum of the probe $\hat{p}_2$ is expected to give information about the generalized position of the signal $\hat{s}_1$, the initial uncertainty in the generalized momentum of the probe should be finite and the initial state of the probe should be a superposition of generalized position states. In this case, the exact solution, i.e., Eq. (2), shows that the signal and the probe are actually entangled after the interaction, while the approximated solution, i.e., Eq. (6), suggests that they are disentangled. Therefore, while a subsequent measurement of the probe would actually lead to a reduction in the state of the signal, the adiabatic approximation suggests that the signal is unchanged. The adiabatic approximation, therefore, is not valid in the analysis of the quantum measurement process of a single system.

In a recent Rapid Communication [10] we considered a measurement scheme in which the signal and the probe are left exactly disentangled after their interaction. The analog to the protected state in our scheme is the squeezed harmonic oscillator state, i.e., an eigenstate of the operator $e^r\hat{s}_1+ie^{-r}\hat{s}_2$, where $\hat{s}_1$ and $\hat{s}_2$ are the generalized position and momentum of the measured harmonic oscillator (the signal). *A priori* knowledge of r, the squeezing parameter of the signal, is used in preparing a probe in a squeezed vacuum state with the *opposite* squeezing, to avoid entanglement of the probe and the signal as they couple linearly. The interaction of the signal and the probe is described by the Hamiltonian $\hat{V}=2\hbar\kappa(\hat{s}_1\hat{p}_1+\hat{s}_2\hat{p}_2)$. Since in this case the signal and the probe are exactly disentangled after their interaction, a subsequent measurement of the probe would yield information about the state of the signal without affecting the signal at all. A measurement of the generalized position (momentum) of the probe $\hat{p}_1$ ($\hat{p}_2$) would give information about the generalized momentum (position) of the signal $\hat{s}_2$ ($\hat{s}_1$). A measurement of the energy of the probe $\hat{p}_1^2+\hat{p}_2^2$ would give information about the energy of the signal $\hat{s}_1^2+\hat{s}_2^2$. In this scheme, in fact, one can measure all of the observables associated with the signal, where in each measurement the entanglement of the signal with the probe is avoided exactly. The price one pays for the exact disentanglement is a deterministic change in the energy of the signal. For the signal to be back exactly in its initial state (up to a phase factor only), one needs to drive the signal back to its initial excitation, and for this purpose one needs to know this initial excitation. We concluded that it is possible to devise a measurement without entanglement for any observable with only partial knowledge of the measured state (in our case, one only needs to know the squeezing parameter r, or noise distribution, of the squeezed state). In a measurement without entanglement, the reduction in the state of the signal, due to a measurement of the probe, is avoided; therefore, there is no stochastic change in the state of the signal. Yet, in order to avoid any change in the state of the signal, i.e., in order to also avoid a deterministic change in the state of the signal due to the unitary interaction with the probe, one needs to have full *a priori* knowledge of this state (in our case, the additional information needed is the initial excitation of the squeezed state, which, together with the initial squeezing parameter of the state comprise a full definition of the initial squeezed state).

To conclude, the adiabatic approximation used in the analysis of the adiabatic protective measurement is not valid for the analysis of the quantum measurement process of a single system because it approximates an actual entanglement of the quantum signal with the quantum probe as a disentanglement and therefore neglects the reduction in the state of the measured system. This reduction can be avoided when the entanglement of the signal with the probe is exactly avoided, using partial *a priori* information on the state of the signal. In this case, the unitary interaction of the signal with the probe would change the state of the signal in a deterministic way. In order to avoid all possible changes in the state of the signal while it is being measured, full *a priori* information about this state is required.

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